

Shear Viscosity in a Perturbative Quark-Gluon-Plasma

John Fuini III^{1,2,‡}, Nasser S. Demir² Dinesh K. Srivastava³ and Steffen A. Bass²

¹ The University of Texas at San Antonio, San Antonio, Texas 78249-1644, USA

² Department of Physics, Duke University, Durham, North Carolina 27708-0305, USA

³ Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700 064, India

E-mail: john.fuini@gmail.com

Abstract. Among the key features of hot and dense QCD matter produced in ultra-relativistic heavy-ion collisions at RHIC is its very low shear viscosity, indicative of the properties of a near-ideal fluid, and a large opacity demonstrated by jet energy loss measurements. In this work, we utilize a microscopic transport model based on the Boltzmann equation with quark and gluon degrees of freedom and cross sections calculated from perturbative Quantum Chromodynamics to simulate an ideal Quark-Gluon-Plasma in full thermal and chemical equilibrium. We then use the Kubo formalism to calculate the shear viscosity to entropy density ratio of the medium as a function of temperature and system composition. One of our key results is that the shear viscosity over entropy-density ratio η/s becomes invariant to the chemical composition of the system when plotted as a function of energy-density instead of temperature.

Submitted to: *J. Phys. G: Nucl. Phys.*

[‡] current address: Department of Physics, University of Washington, Seattle, WA 98195-1560

1. Introduction

Ultrarelativistic heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) are thought to have produced a Quark Gluon Plasma (QGP) with the characteristics of a near ideal fluid [1, 2, 3, 4]. One of the most important current challenges in QGP research is to quantify the transport coefficients of this novel state of matter. Recently, attention in the field has been primarily focused on the shear viscosity to entropy density ratio η/s . Calculations utilizing certain strongly coupled supersymmetric gauge theories with gravity duals [5] postulate a lower bound of $\eta_{\min} = s/4\pi$ for this quantity, often referred to as the KSS bound [6]. Relativistic viscous hydrodynamic calculations have confirmed very low values of η/s in order to reproduce the RHIC elliptic flow (v_2) data [7, 8, 9]. However, current calculations assume a fixed value of η/s throughout the entire evolution of the system and neglect its temperature dependence, which anyhow cannot be obtained from the AdS/QCD calculations.

It has been argued on very general grounds that η/s should exhibit a minimum at the phase-transition and should rise for decreasing temperature in the hadronic phase and for increasing temperature in the deconfined phase [10]. Recent calculations of η/s using microscopic transport theory in the hadronic sector have confirmed this expectation and have provided quantitative guidance with respect to the temperature dependence of η/s in and out of chemical equilibrium [11]. At temperatures above T_C there remains a large uncertainty regarding the value and temperature-dependence of η/s : in the limit of very high temperatures, the interactions between the constituents of QCD matter should be calculable with perturbative methods [12], whereas in the strongly interacting regime near T_C η/s could be close to the KSS bound. It is our goal in this paper to explore the temperature dependence of QCD matter for temperatures above T_C at which quasi-particle degrees of freedom become viable. For our calculation we will use a microscopic transport model, the BMS implementation of the Parton Cascade Model (PCM) [13], which has the advantage that it can describe both, out of equilibrium QCD matter created in collisions of ultra-relativistic heavy-ions, as well as equilibrated QCD matter at a fixed temperature. For the calculation of the shear viscosity we shall use the Kubo formalism utilized in our previous calculation of η and η/s in the hadronic sector [11].

2. The Parton Cascade Model

The Parton Cascade Model (PCM) [14, 15] is a microscopic transport model which is used to simulate the time evolution of a system of quarks and gluons utilizing the Boltzmann equation. Here, we use the BMS implementation, which has been applied successfully to the calculation of direct photon production [16, 17] and baryon stopping [18] at RHIC. PCM implementations by other groups have been used to address collective flow, parton energy-loss as well as multi-particle effects on transport coefficients [19, 20, 21, 22, 23, 24, 25, 26]. Our calculation focuses on an ideal Quark-

Gluon-Plasma, i.e. a gas of u, d and s quarks and anti-quarks as well as gluons at fixed temperature T in full thermal and chemical equilibrium. In addition, we also study a one-component gluon plasma in thermal and chemical equilibrium. For our transport calculation we define a box with periodic boundary conditions (to simulate infinite matter) and sample thermal quark and gluon distribution functions to generate an ensemble of particles at a given temperature and zero chemical potential.

The time evolution of our system is described by a Boltzmann transport equation:

$$p^\mu \frac{\partial}{\partial x^\mu} F_k(x, \vec{p}) = \sum_{\text{processes: } i} \mathcal{C}_i[F] \quad (1)$$

where the collision terms \mathcal{C}_i are nonlinear functionals of the phase-space distribution function. Although the collision term, in principle, includes factors encoding the Bose-Einstein or Fermi-Dirac statistics of the partons, we neglect those effects here.

The collision integrals have the form:

$$\mathcal{C}_i[F] = \frac{(2\pi)^4}{2S_i} \cdot \int \prod_j d\Gamma_j |\mathcal{M}_i|^2 \delta^4(P_{\text{in}} - P_{\text{out}}) D(F_k(x, \vec{p})) \quad (2)$$

with

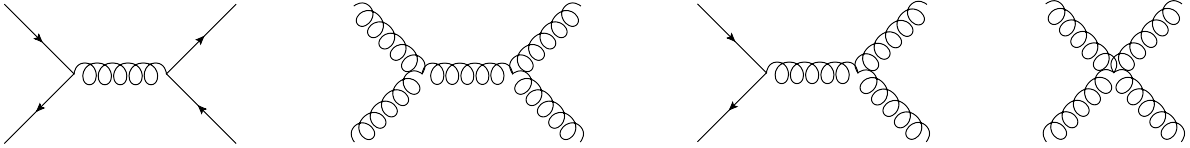
$$D(F_k(x, \vec{p})) = \prod_{j=\text{out}} F_j(x, \vec{p}) - F_k(x, \vec{p}) \prod_{\substack{j=\text{in} \\ j \neq k}} F_j(x, \vec{p}) \quad (3)$$

and

$$\prod_j d\Gamma_j = \prod_{\substack{j \neq k \\ \text{in, out}}} \frac{d^3 p_j}{(2\pi)^3 (2p_j^0)} \quad (4)$$

S_i is a statistical factor defined as $S_i = \prod_{j \neq k} K_a^{\text{in}}! K_a^{\text{out}}!$ with $K_a^{\text{in, out}}$ identical partons of species a in the initial or final state of the process i , excluding the k th parton.

The matrix elements for the full Quark-Gluon-Plasma calculation $|\mathcal{M}|^2$ account for the following processes (note that all t and u -channel processes are included as well):



The corresponding scattering cross sections are expressed in terms of spin- and color-averaged amplitudes $|\mathcal{M}|^2$:

$$\left(\frac{d\hat{\sigma}}{dQ^2} \right)_{ab \rightarrow cd} = \frac{1}{16\pi \hat{s}^2} \langle |\mathcal{M}|^2 \rangle \quad (5)$$

In the case of a pure Gluon Plasma (GP), we use the following differential cross section in order to facilitate comparisons with PCM implementations by other groups [19, 20, 21, 22, 23]:

$$\frac{d\sigma^{gg \rightarrow gg}}{dQ^2} = 2\pi \alpha_s^2 \frac{9}{4} \frac{1}{(Q^s + m_D^2)^2}, \quad (6)$$

For the transport calculation we also need the total cross section as a function of \hat{s} which can be obtained from (5):

$$\hat{\sigma}_{ab}(\hat{s}) = \sum_{c,d} \int_0^{\hat{s}} \left(\frac{d\hat{\sigma}}{dQ^2} \right)_{ab \rightarrow cd} dQ^2 \quad . \quad (7)$$

The amplitudes for the above processes have been calculated in refs. [27, 28] for massless quarks. Note, however, that for the kinematics of the system, the light quark masses are treated explicitly. We regularize the IR divergence of the cross sections with a temperature dependent Debye mass m_D . We shall use two different expressions for m_D – the first one is a Debye-mass for particles obeying Boltzmann statistics which has been used in [21, 23, 24, 25, 26] and which we utilize to allow our results to be compared to these calculations:

$$m_D(T) = \sqrt{\frac{24}{\pi} \alpha_s T^2} \quad , \quad (8)$$

and the second one is the standard Debye mass used in pQCD calculations for systems of quarks and gluons:

$$m_D(T) = gT \sqrt{(2N_c + N_f)/6} \quad (9)$$

The first m_D parametrization we shall refer to as Boltzmann- m_D whereas the second parametrization we shall refer to as regular m_D . In both cases the coupling constant is defined as

$$\alpha_s = \frac{g^2}{4\pi} \quad (10)$$

and can either be chosen as a constant parameter, or to have the following temperature dependence [10]:

$$\frac{1}{g^2} = \frac{9}{8\pi^2} \ln \left(\frac{T}{\Lambda_T} \right) + \frac{4}{9\pi^2} \ln \left(2 \ln \left(\frac{T}{\Lambda_T} \right) \right) \quad (11)$$

with $\Lambda_T = 30$ MeV.

3. Methodology

3.1. Shear-viscosity

In order to calculate the shear-viscosity of our system, we employ the Kubo-formalism [29, 30]. The methodology for applying the Kubo-formalism to infinite QCD matter modeled by microscopic transport theory was developed in [31] and has already been successfully applied to calculating the shear viscosity for a hadron gas [31, 11]. The momenta of all the partons in our system are tracked over the course of 20 time-steps of 0.5 fm/c. Knowing the momenta p of all partons in the system, the discretized stress-energy tensor of our system can be calculated at each timestep:

$$\pi^{\mu\nu} = \int d^3p \frac{p^\mu p^\nu}{p^0} f(x, p) \Rightarrow \frac{1}{V} \sum_{i=1}^N \frac{p^x(i) p^y(i)}{p^0} \quad (12)$$

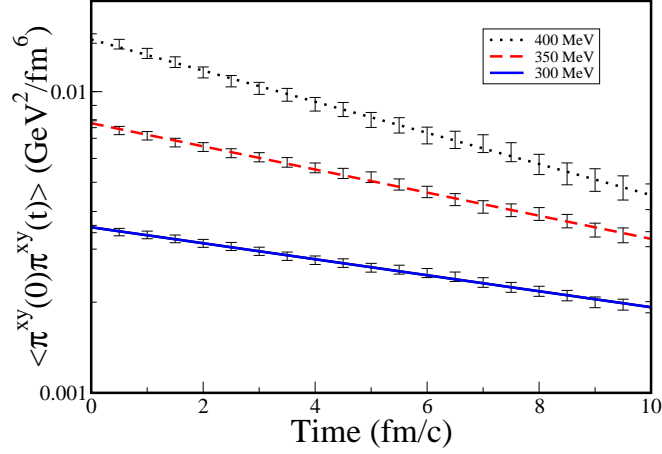


Figure 1. Kubo correlators for a system of gluons at various temperatures. The correlators show a clear exponential decay.

The stress-energy tensor now allows for the construction of the stress energy tensor correlator used in the Kubo formalism [29, 30] to calculate the shear viscosity via

$$\eta = \frac{1}{T} \int d^3r \int_0^\infty dt \langle \pi^{xy}(0,0) \pi^{xy}(\vec{r}, t) \rangle_{\text{equil}} \quad (13)$$

where the average is taken over many events and T is the temperature of the system. A selection of correlators for different temperatures are shown in figure 1. Based on the observed exponential decay of the correlators, we fit these with the following analytic function:

$$\langle \pi^{xy}(0,0) \pi^{xy}(\vec{r}, t) \rangle \sim e^{-\frac{t}{\tau_R}} \quad (14)$$

with the parameter τ_R called the relaxation time of the system. Inserting this function into the expression for η yields:

$$\eta = \frac{V}{T} \langle \pi^{xy}(0,0) \pi^{xy}(0,0) \rangle \tau_R \quad (15)$$

with the volume V and temperature T being input parameters of our calculation and τ_R and $\langle \pi^{xy}(0,0) \pi^{xy}(0,0) \rangle$ determined by our exponential fit (see figure 1).

3.2. Entropy

The entropy-density of our system is being calculated via the Gibbs entropy density:

$$s = \frac{\epsilon + P + \mu_B \rho_B}{T} \quad (16)$$

Since the system is initialized with net baryon density, $\rho = 0$, the pressure P and energy-density ϵ are the only quantities which need to be extracted from our simulation according to

$$P = \frac{1}{3V} \sum_{i=1}^N \frac{|\vec{p}(i)|^2}{p^0(i)} \quad \text{and} \quad \epsilon = \frac{1}{V} \sum_{i=1}^N p^0(i) \quad . \quad (17)$$

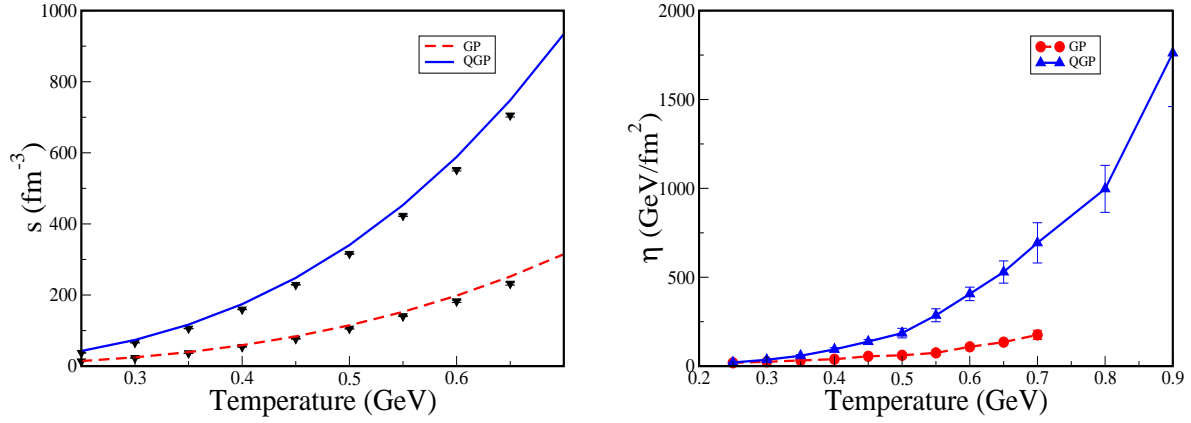


Figure 2. Left: entropy as a function of temperature for a Gluon-Plasma and a three flavor Quark-Gluon-Plasma. The symbols denote the entropy of the system evaluated using the Gibbs formula and the lines represent the Stefan Boltzmann expression for the entropy density with $\nu(T) = 16$, and Quark-Gluon-Plasma, $\nu(T) = 47.5$ Right: shear-viscosity η as a function of temperature for the Gluon Plasma (GP) and the Quark-Gluon-Plasma (QGP).

The left frame of figure 2 shows the resulting entropy-density for a Gluon Plasma (GP) as well as a Quark-Gluon-Plasma (QGP) containing gluons and three light quark flavors.

We can compare the entropy density directly obtained from our calculation to the Stefan Boltzmann entropy density given by:

$$s = \frac{2\pi^2}{45}\nu(T)T^3 \quad (18)$$

where $\nu(T) = N_b + \frac{7}{8}N_f$ and N_b and N_f are the bosonic and fermionic degrees of freedom. The results of the comparison are shown in the left frame of and figure 2 and show good agreement between the Stefan Boltzmann entropy and our system entropy calculated via the Gibbs relation.

4. Results and Discussion

We will present our results predominantly for two modes of calculation: the first one, denoted by GP, is for a gluon plasma with a regular Deybe screening mass and temperature dependent coupling. The second mode, denoted by QGP, uses the same temperature-dependent parametrizations for m_D and the coupling constant, but is for a quark-gluon plasma with three light quark flavors. We consider the QGP mode to be the most realistic mode of calculation presented here.

The right frame of Figure 2 displays the results of our calculation for the shear viscosity for a system of gluons as well as a system of gluons and three light quark flavors. Both, the shear viscosity, as well as the entropy density depicted in the left frame of figure 2, rise strongly as a function of increasing temperature. It is interesting to note that the QGP shows a significantly higher shear-viscosity than the GP for a given temperature, which seems counter-intuitive given the larger particle density of the QGP,

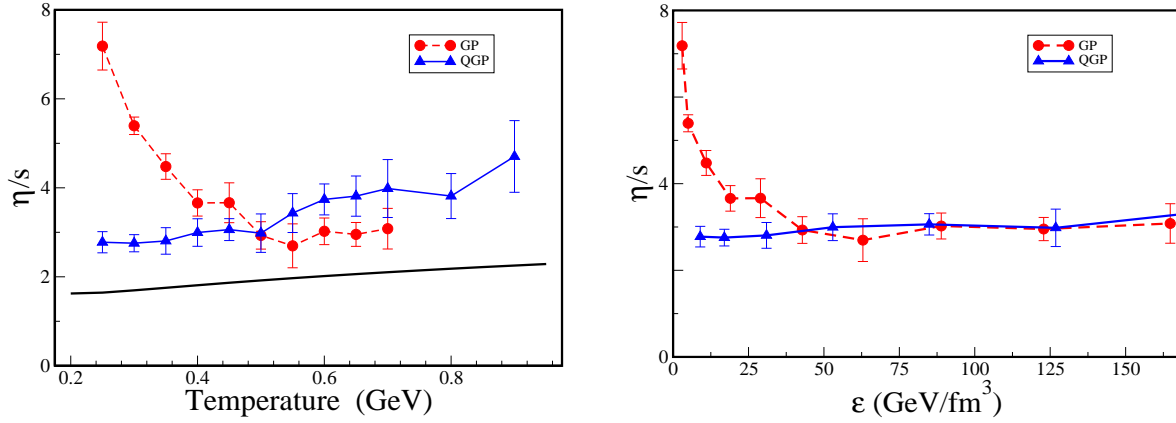


Figure 3. Left: η/s as a function of temperature for a Gluon-Plasma (full circles) and Quark-Gluon-Plasma (full triangles), compared to a HTL calculation (solid line) [12]. Right: same result, but plotted vs. energy density.

but is probably due to the smaller interaction cross-sections among the quarks of the system.

Having calculated both, the shear-viscosity as well as the entropy-density of our system, we can now turn to the ratio η/s , made famous by the KSS bound: the left frame of figure 3 shows η/s as a function of temperature for the GP (full circles) and QGP (full triangles) case, compared to an analytic HTL calculation of a three-flavor QGP [12] (solid line). The QGP calculation of η/s shows a monotonous rise as a function of temperature with a slope very similar to that of the HTL calculation. The differences in absolute value between the two can be understood by considering the NLL corrections present in the HTL calculation into account, which are missing in our simulation of QCD matter. The Gluon Plasma exhibits an upturn in η/s for temperatures below 500 MeV – we attribute this unexpected rise towards lower temperatures to a breakdown in the perturbative approximations present in our calculations, emphasized by taking the ratio of two quantities, which both, when calculated and viewed separately (see figure 2), seem to exhibit a smooth behavior in the low temperature domain below 500 MeV.

Comparing η/s of a GP and a QGP at the same temperature may be misleading due to the significantly larger parton density present in a QGP. Therefore in the right frame of figure 3 we compare η/s for the two scenarios at equivalent energy-density and find for energy-densities above 35 GeV/fm^3 excellent agreement between the two systems. This is of particular relevance since the flavor composition of the deconfined QCD matter created in ultra-relativistic heavy-ion collisions is by no means fully established and may change strongly as a function of time – from a gluon-dominated system being created by the decay of a Color-Glass-Condensate to a QGP in full thermal and chemical equilibrium as the system progresses in its hydrodynamic expansion. Our result indicates that η/s , a quantity which controls the hydrodynamic evolution of the system, should be fairly robust with respect to its flavor composition when taken as a function of energy-density instead of temperature.

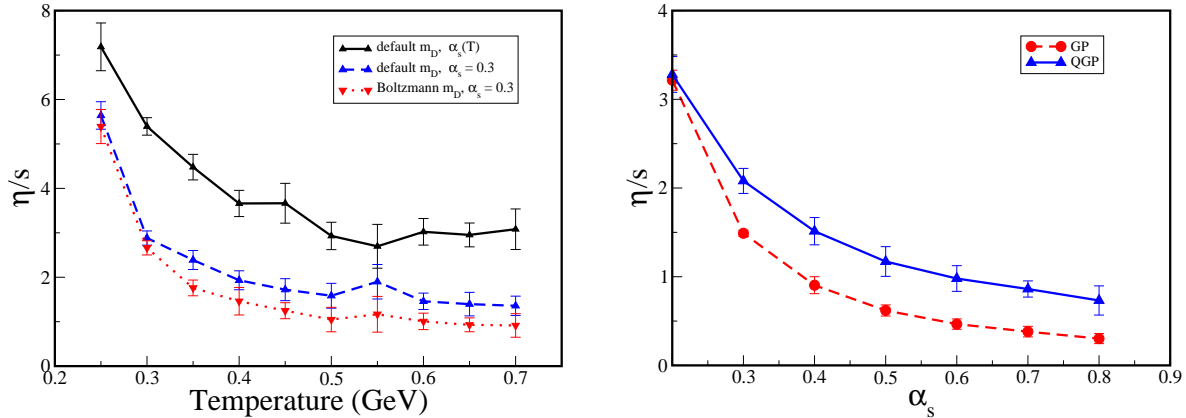


Figure 4. Left: effect of different Debye mass parametrizations and temperature-dependent coupling on η/s . Right: Effect of coupling constant on η/s .

In the left frame of figure 4 we study the effects of our different Debye-mass parametrizations and the temperature-dependence on η/s . For this purpose we calculate η/s for a gas of gluons with a Boltzmann Debye-mass at fixed coupling of $\alpha_s = 0.3$ (i.e. the same system as e.g. in [21, 22, 23, 24, 25]) and compare this calculation to one with our default parametrization (taken with $N_c = 3$ and $N_f = 0$). We find the effect of the different m_D parametrizations to be small, on the 10% to 15% level, with the Boltzmann Debye-mass giving systematically smaller values of η/s . If we now replace the fixed coupling with a temperature dependent coupling constant, the value of η/s increases roughly by a factor of 2.

So far we have restricted our investigation to a purely perturbative partonic system with the respective temperature-dependent screening masses and coupling strengths. However, there are strong indications that the medium created in ultra-relativistic heavy-ion collisions is non-perturbative in nature – at the very least within the temperature range from T_C to approximately $3 - 4T_C$, which is covered by our calculations. One method to explore the behavior of η/s at stronger coupling is to treat the coupling constant in our calculations as a free parameter and then study η/s at fixed temperature as a function of the coupling constant. The right frame of figure 4 shows η/s as a function of coupling strength for the gluon plasma and the quark-gluon-plasma. In the strong coupling limit, in particular for the gluon plasma, values of $\eta/s \ll 1$ can be obtained, yielding results compatible with a fluid-dynamical analysis of RHIC data [9]. Similar results have been obtained by [20], who directly increased the gluon-gluon scattering cross-section, which is equivalent to an increase in the coupling constant or by directly studying η/s as a function of the coupling constant [22]. One should note, however, that treating the coupling constant as a free parameter yields a medium with inconsistent characteristics, since the particle density and screening mass are still controlled by the temperature, whereas the coupling is not. Also, for large couplings $\alpha_s \simeq 1$ the perturbative assumptions underlying the PCM are not valid anymore.

The final question we wish to address is the effect the angular distribution of

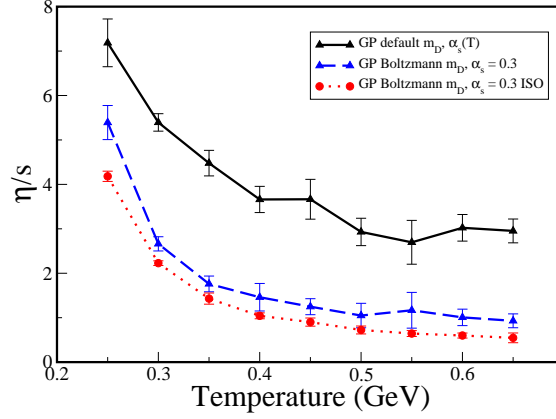


Figure 5. Coupling constant and angular distribution dependence of η/s for a gluon plasma: The choice of a fixed coupling constant affects η/s to a far larger degree than the transition from the regular angular distribution to isotropic scattering.

the scattering partons, i.e. the exact form of the differential matrix-element, has on η/s : figure 5 shows a comparison between our default gluon plasma calculation compared with results at fixed coupling and regular matrix elements as well as with the same matrix elements using an isotropic angular distribution. The main effect, a reduction of η/s by a factor of 3, comes from the choice of a fixed coupling constant vs. a temperature dependent one. Changing from forward-backward peaked to isotropic scattering provides an additional 10% - 20% effect, but not a dramatic reduction in η/s . This finding is of particular interest in the context of work done by [21, 22, 23, 24, 25], employing a PCM including radiative corrections (i.e. $2 \rightarrow 3$ and $3 \rightarrow 2$ processes) and an implementation of the LPM effect, which manifests itself in a near-isotropic angular distribution for the third particle in the outgoing channel. It has been shown that the isotropic emission of the radiated particle in this implementation of the LPM effect plays an important role for the thermalization of the system. Our results utilizing solely $2 \rightarrow 2$ scattering processes indicate that it is most likely a combination of multiple effects – choice of fixed coupling constant, the isotropic angular distribution of the LPM implementation, and the inclusion of radiative corrections, which is responsible for the low viscosity findings of these calculations [22, 23].

5. Summary

We have utilized the parton cascade model to simulate a perturbative quark-gluon-plasma in full thermal equilibrium and have extracted its shear-viscosity as well as the shear-viscosity over entropy-density ratio as a function of temperature. We find that our results depend significantly on the details of the calculation, i.e. choice of coupling constant and parametrization of the Debye screening mass as well as the degrees of freedom (gluon vs. quark-gluon plasma). One of our key results is that the shear viscosity over entropy-density ratio η/s becomes invariant to the chemical composition

of the system when plotted as a function of energy-density instead of temperature. The values we obtain for η/s are higher than those expected for the near ideal fluid observed at RHIC; in particular they are not compatible with η/s values extracted from viscous fluid dynamics analysis [9] of elliptic flow data. By increasing the coupling constant we find values of η/s that are compatible with the RHIC data, but only for values of the coupling at which the perturbative assumptions of the PCM may not anymore be valid. Inclusion of quantum-coherence effects, such as the LPM effect, multi-particle scattering processes or turbulent color fields leading to an anomalous viscosity [32, 33] may explain the origin of the observed small η/s values, but the final determination of this question remains to be settled in future work.

6. Acknowledgments

SAB would like to thank Peter Arnold, Carsten Greiner, Berndt Müller and Zhe Xu for helpful discussions. This work was supported by the DOE under grant DE-FG02-05ER41367. JF acknowledges support from the TUNL REU program under NSF award NSF-PHY-08-51813. The calculations were performed using resources provided by the Open Science Grid, which is supported by the National Science Foundation and the U.S. Department of Energy's Office of Science.

References

- [1] Adcox K *et al.* (PHENIX) 2005 *Nucl. Phys.* **A757** 184–283 (*Preprint nucl-ex/0410003*)
- [2] Back B B *et al.* 2005 *Nucl. Phys.* **A757** 28–101 (*Preprint nucl-ex/0410022*)
- [3] Arsene I *et al.* (BRAHMS) 2005 *Nucl. Phys.* **A757** 1–27 (*Preprint nucl-ex/0410020*)
- [4] Adams J *et al.* (STAR) 2005 *Nucl. Phys.* **A757** 102–183 (*Preprint nucl-ex/0501009*)
- [5] Policastro G, Son D T and Starinets A O 2001 *Phys. Rev. Lett.* **87** 081601 (*Preprint hep-th/0104066*)
- [6] Kovtun P, Son D T and Starinets A O 2005 *Phys. Rev. Lett.* **94** 111601 (*Preprint hep-th/0405231*)
- [7] Song H and Heinz U W 2008 *Phys. Lett.* **B658** 279–283 (*Preprint 0709.0742*)
- [8] Romatschke P and Romatschke U 2007 *Phys. Rev. Lett.* **99** 172301 (*Preprint 0706.1522*)
- [9] Luzum M and Romatschke P 2008 *Phys. Rev.* **C78** 034915 (*Preprint 0804.4015*)
- [10] Csernai L P, Kapusta J I and McLerran L D 2006 *Phys. Rev. Lett.* **97** 152303 (*Preprint nucl-th/0604032*)
- [11] Demir N and Bass S A 2009 *Phys. Rev. Lett.* **102** 172302 (*Preprint 0812.2422*)
- [12] Arnold P, Moore G D and Yaffe L G 2003 *JHEP* **05** 051 (*Preprint hep-ph/0302165*)
- [13] Bass S A, Muller B and Srivastava D K 2003 *Phys. Lett.* **B551** 277–283 (*Preprint nucl-th/0207042*)
- [14] Geiger K and Muller B 1992 *Nucl. Phys.* **B369** 600–654
- [15] Geiger K 1997 *Comput. Phys. Commun.* **104** 70–160 (*Preprint hep-ph/9701226*)
- [16] Bass S A, Muller B and Srivastava D K 2003 *Phys. Rev. Lett.* **90** 082301 (*Preprint nucl-th/0209030*)
- [17] Renk T, Bass S A and Srivastava D K 2006 *Phys. Lett.* **B632** 632–637 (*Preprint nucl-th/0505059*)
- [18] Bass S A, Muller B and Srivastava D K 2003 *Phys. Rev. Lett.* **91** 052302 (*Preprint nucl-th/0212103*)
- [19] Molnar D and Gyulassy M 2000 *Phys. Rev.* **C62** 054907 (*Preprint nucl-th/0005051*)
- [20] Molnar D and Huovinen P 2005 *Phys. Rev. Lett.* **94** 012302 (*Preprint nucl-th/0404065*)

- [21] Xu Z and Greiner C 2005 *Phys. Rev.* **C71** 064901 (*Preprint hep-ph/0406278*)
- [22] Xu Z and Greiner C 2008 *Phys. Rev. Lett.* **100** 172301 (*Preprint 0710.5719*)
- [23] El A, Muronga A, Xu Z and Greiner C 2009 *Phys. Rev.* **C79** 044914 (*Preprint 0812.2762*)
- [24] Fochler O, Xu Z and Greiner C 2009 *Phys. Rev. Lett.* **102** 202301 (*Preprint 0806.1169*)
- [25] Xu Z and Greiner C 2009 *Phys. Rev.* **C79** 014904 (*Preprint 0811.2940*)
- [26] Uphoff J, Fochler O, Xu Z and Greiner C 2010 (*Preprint 1004.4091*)
- [27] Cutler R and Sivers D W 1978 *Phys. Rev.* **D17** 196
- [28] Combridge B L, Kripfganz J and Ranft J 1977 *Phys. Lett.* **B70** 234
- [29] Kubo R 1957 *J. Phys. Soc. Jap.* **12** 570–586
- [30] Kubo R 1966 *Rept. Prog. Phys.* **29** 255
- [31] Muronga A 2004 *Phys. Rev.* **C69** 044901 (*Preprint nucl-th/0309056*)
- [32] Asakawa M, Bass S A and Muller B 2006 *Phys. Rev. Lett.* **96** 252301 (*Preprint hep-ph/0603092*)
- [33] Asakawa M, Bass S A and Muller B 2007 *Prog.Theor.Phys.* **116** 725–755 (*Preprint hep-ph/0608270*)